Spectra of Hydrogen atom and its fine structure

(1) 1S

In case of 1S electron;

$l = 0$

$S = 1/2$

$j = l + s = 0 + 1/2 = 1/2$

Spin-orbit interaction energy:

ΔE_{SO}

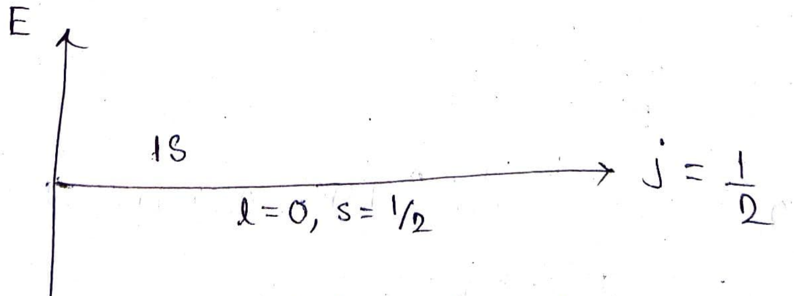
$$= \frac{a}{2}\left[j(j+1) - l(l+1) - s(s+1)\right]$$

where $a = \frac{\alpha Z \hbar^3}{4 m^2 c r^3}$; α being fine-structure constant.

$\therefore \Delta E_{SO}$

$$= \frac{a}{2}\left[\frac{1}{2}\left(\frac{1}{2}+1\right) - 0(0+1) - \frac{1}{2}\left(\frac{1}{2}+1\right)\right]$$

$$= \frac{a}{2} \times 0 = 0.$$



$\therefore$ There is no splitting due to spin-orbit coupling.

(2) 2s

(3) 3s

→ Both (2) and (3) are similar as in the case of 1s electron.

(4.) 3d

→ In case of 3d electron;

$l = 2$

$s = \frac{1}{2}$

$j = l \pm s = \frac{5}{2}, \frac{3}{2}$

Spin-orbit interaction energy;

$$\Delta E_{SO} = \frac{a}{2}\left[j(j+1) - l(l+1) - s(s+1)\right]$$

For $j = \frac{5}{2}$

$$\Delta E_{SO} = \frac{a}{2}\left[\frac{5}{2}\left(\frac{5}{2}+1\right) - 2(2+1) - \frac{1}{2}\left(\frac{1}{2}+1\right)\right]$$

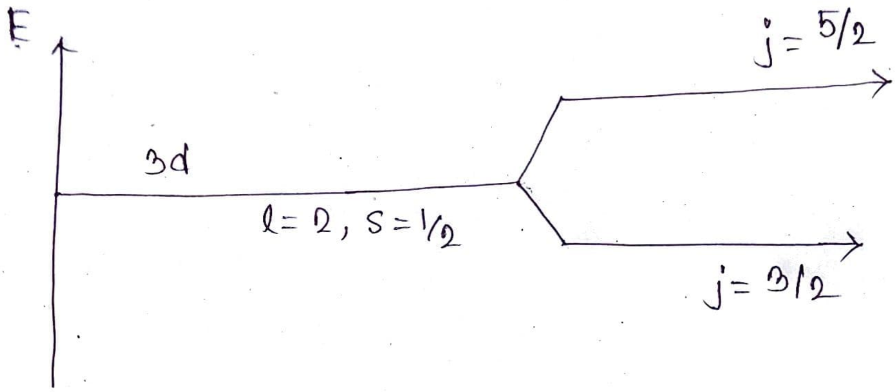
$$= \frac{a}{2} \times 2$$

$$= a$$

For $j = \frac{3}{2}$

$$\Delta E_{SO} = \frac{a}{2}\left[\frac{3}{2}\left(\frac{3}{2}+1\right) - 2(2+1) - \frac{1}{2}\left(\frac{1}{2}+1\right)\right]$$

$$= \frac{a}{2}(-3) = -\frac{3}{2}a$$



This splitting is due to spin-orbit coupling or spin orbit interaction.